Week 9 Notes
Astro 2 (Discussion Section 101)

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Administrative Tasks

Evaluations At the end of class, I'll pass out evaluations. I'll need someone to take them back to Broida.

Midterm 2 Midterm 2 is back. I'll hand it back before we do evaluations.

Review

Midterm 2 Wrap-up

I'll run through the solutions to Midterm 2.

Heisenberg Uncertainty Principle

One of the most interesting principles that came from quantum mechanics is the Heisenberg Uncertainty Principle. In its most basic form, it places limits on how accurately we can know both the position and momentum of a particle. Mathematically, this has the form

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$

Here Δx is the uncertainty we have in the particle's position and Δp is the uncertainty in the particle's momentum. This isn't an experimental limit; *we will never be able to be more accurate*. This gives rise to a lot of philosophical arguments of what it means to "measure" a particle. One relatively simple way to answer it is to point out that if we had some perfect way to measure the momentum of a particle but then wanted to know where it is, we would (in theory) shine light on it. However, the photons of light would impart their momentum on the particle, affecting its momentum. Every measurement of position comes with a price on the measurement of the momentum and vice versa.

For the philosophers in the class, this principle in many ways defeats the idea of a deterministic universe. A deterministic universe is one in which at some point in time, we knew the position and momentum of every single particle in the universe. In theory, we could then predict everything that would ever happen. However, since the Heisenberg Uncertainty Principle forbids us knowing this, the universe it not deterministic. The question that still remains is whether or not every particle even *has* a definite position and momentum at any given time. This gets into the more nitty gritty of quantum mechanics, so I won't go into it for now.

However, we *are* interested in another form of the Heisenber uncertainty principle, which states that the uncertainty in energy of a system over a duration of time is also limited by this value of $h/2\pi$. Mathematically, this has the form

$$\Delta E \, \Delta t \geq \frac{h}{2\pi}$$

Where ΔE is the uncertainty in energy that could be measured over a time duration Δt. Thus we see that over extremely short time scales, (small Δt), the energy can fluctuate *dramatically* (large ΔE). This is what allows the production of **virtual pairs**. A large amount of energy is borrowed from the vacuum to create a particle-antiparticle pair that must quickly annihilate so that the energy debt is paid before the corresponding Δt elapses.

Example: What's the longest time interval under which a virtual pair of proton and anti-proton might spontaneously exist?

To create the pair, an energy equal to the sum of the rest energies must be paid out from the vacuum. Recall then that $E_0 = m_p c^2$. So then

$$\Delta E = 2m_p c^2 = 2(1.67 \times 10^{-27}\text{ kg})(3 \times 10^8\text{ m/c})^2 \approx 3 \times 10^{-10}\text{ J}$$

Then the maximum amount of time is when then inequality in the uncertainty principle is an equality:

$$\Delta E\,\Delta t = \frac{h}{2\pi} \Rightarrow \Delta t = \frac{h}{2\pi \Delta E} = \frac{6.626 \times 10^{-34}\text{ J s}}{2\pi(3 \times 10^{-10}\text{ J})} = 3.52 \times 10^{-25}\text{ s}$$

What does this mean? It means that on the time scale of 3.52×10^{-25} s, proton-antiproton pairs can freely come into existence and annihilate without violating any laws of physics.

The Jeans Length

Newton's Proposal Suppose you have a gas that you have a gas with only very tiny fluctuations in density. These regions will attract more material from nearby. However, as the "clump" grows, so does the gas pressure nearby, which would help repel gas. The question, then, is when does gravity overwhelm gas pressure to allow the creation of a "permanent" structure.

Jeans' Solution The British physicist James Jeans solved this problem. He found that fluctuations that extend over lengths greater than a certain length (called the **Jeans Length**) will result in objects forming. The mathematical form of the Jeans length is

$$L_{\mathrm{J}} = \sqrt{\frac{\pi k T}{m G \rho_m}}$$

where k is the Boltzmann constant (1.38×10^{-23} J/K), T is the ambient temperature, m is the mass of a single particle of the gas, G is the gravitational constant, and ρ_m is the average density of matter in the gas.

Fluctuations larger than the Jeans length tend to grow and those smaller than the Jeans length will fade away. This concept is important in understanding how the early universe developed its current structure. We use this technique to surmise that globular clusters were among the first large-scale structures to form in the universe.

Test Returns and Evaluations