Week 7 Notes
Astro 2 (Discussion Section 101)

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Administrative Tasks

Sargazing

Remember that our stargazing night will take place **tonight** (May 9). We'll meet at 9:00 in front of the elevators in Broida. The event should last no more than an hour and is not for extra credit.

Midterm 2 reminder

Remember that midterm 2 is scheduled for Monday, May 17, so next week will be another review session. Come with questions!

Midterm 1 is back!

I'll hand Midterm 1 back to students and explain the curve at the end of class.

Review

A Quick Note on Problem 1

I'll mention a quick hint on homework problem 1 for those who attend section.

Expansion of the Universe

We know from Hubble's Law that the Universe is expanding but what does that mean for what we observe in the universe? When we look far away, we are looking back in time, so will we see things denser or sparser than they actually are today? Is space hotter or colder than it is today?

Recall the definition of redshift:

$$z = \frac{\lambda - \lambda_0}{\lambda_0}$$

Simplifying the fraction, we get

$$z = \frac{\lambda}{\lambda_0} - 1 \Rightarrow z + 1 = \frac{\lambda}{\lambda_0}$$

So $z+1$ gives us our "stretching factor" wavelengths are stretched to a length $z+1$ times what they originally were. As it turns out, this factor, $z+1$ pretty much uniquely scales all dimensions when looking back in time. For instance, if we observe a chunk of space that was 1 meter long in our telescope at redshift $z = 1$, it is currently

$$L_0 = (z+1)L = 2(1\text{ m}) = 2\text{ m}$$

so in its own reference frame right now, that chunk of space is now 2 meters long. When doing calculations like this, be sure that you put the factors of $(z+1)$ in the right places. Remember, the dimensions we measure in our telescope are always the dimensions from the *past*, which I denote with regular variables. The *actual* dimensions right *now* cannot be measured. I denote these with a zero on the variable (you can make up your own system, there is no standard to my knowledge). Note also that in this example I didn't say we were observing a meter stick. A meter stick would not expand in time (why?) the way space would. This is because electromagnetic forces hold the stick together, causing it to not expand with the Hubble flow.

Example Suppose we observe a square of space with sides of length 1 pc at a $z = 3$. How much bigger is the area now than it was when the light we are receiving was first emitted?

Letting A be the area now and A' be the original area, we see that we observe a measured area of $A = 1$ pc^2. Since we know individual lengths scale as $(1+z)$ we may scale each side of the square:

$$s_0 = (z+1)s$$

To get the area being

$$A_0 = (z+1)^2 s^2 = (z+1)^2 A = 16A$$

So the current area of space is $A_0/A = 16$ times larger than how we currently view it. Note that this process works for volume as well, meaning that

$$V_0 = (z+1)^3 V$$

Discussion: Universal Geometries

In class earlier we talked about three possibilities for geometries of the universe and their analogues in two dimensions:

- Positive curvature / Closed, finite, parallel light beams converge (surface of sphere)
- Zero curvature / Flat, infinite, parallel light beams remain parallel (infinite plane)
- Negative curvature / Open, infinite, parallel light beams diverge (saddle-shaped)

In the closed universe, initially parallel light beams must converge, where as in an open universe they must diverge (go away from each other). In the infinite models, the universe has always been infinite, but it is also expanding. Current tests show that the universe is probably flat, or at least very close. Questions?

Density and Density Parameters

By Einstein's famous equation, $E = mc^2$, we can relate the energy from photons to an effective mass, or even a density. Combining the Stefan-Boltzmann law with $E = mc^2$ yields

$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3}$$

where σ is the Stefan-Boltzmann constant, T is the temperature of the radiation, in Kelvin, and c, as usual, is the speed of light. One could compute this for the universe using the cosmic microwave background radiation temperature of $T = 2.725$, since most of the universe's electromagnetic radiation is in the CMB. The result, as quoted in the text, is $\rho_{\text{rad}} = 4.6 \times 10^{-31}$ kg/m^3. One can also compute an approximate matter density of the universe by observing the mass content of the universe. This is done by observing gravitational effects over large spaces to deduce the matter content and then dividing by the volume. We have an approximate measurement of $\rho_{\text{m}} = 2.4 \times 10^{-27}$ kg/m^3. This has an uncertainty of approximately 15%.

Density Parameters The density of the universe actually defines what geometry it takes on. If the total density is above some critical density, it is closed. This density is

$$\rho_{\text{c}} = \frac{3H_0^2}{8\pi G} = 1.0 \times 10^{-26} \text{ kg}$$

This is really the only relevant density, we often express the mass and radiation densities in terms of density parameters, which is the ratio of each density to the critical density:

$$\Omega_{\text{m}} = \frac{\rho_{\text{m}}}{\rho_{\text{c}}} = 0.24$$

$$\Omega_{\text{rad}} = \frac{\rho_{\text{rad}}}{\rho_{\text{c}}} = .000046$$

If we let ρ_0 be the total combined average density, that is the density due to matter, radiation, and any other forms of energy, we may define the **density parameter**:

$$\Omega_0 \equiv \frac{\rho_0}{\rho_\mathrm{c}}$$

We see then that if $\Omega_0 > 1$, we live in a closed universe. If $\Omega_0 = 1$, we live in a flat universe, and if $\Omega_0 < 1$, we live in an open universe. Scientists have studied the propagation of photons that have traveled vast distances (and thus their deviations from being parallel would be quite pronounced) and concluded that the universe is flat or very close to being flat. However, this is a problem, since our current value for the density is too small, just including matter and radiation:

$$\Omega_0 = \Omega_\mathrm{m} + \Omega_\mathrm{rad} = 0.24 < 1$$

We would expect to see an open universe. The "solution" to this problem, though not well understood at all, is the elusive **dark energy**. This gives rise to the **dark energy density parameter**:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_\mathrm{c}}$$

Since ρ_rad is so small, we may approximate $\rho_0 = \rho_\mathrm{m} + \rho_\Lambda$, which then gives us

$$\Omega_0 = \Omega_\mathrm{m} + \Omega_\Lambda$$

We see then, by the assumption that the universe flat, or very close to flat, that $\Omega_\Lambda \approx 0.76$ in order to force $\Omega_0 = 1$.

Evolution of the Density Parameters The density parameters must necessarily change in time. For instance, the universe used to be dominated by radiation but this is no longer the case. Clearly Ω_rad was once greater than Ω_m. Since these Ω_m is dependent on the volume of interest, it must necessarily scale as $(z+1)^3$. Back in time, volumes were smaller, so it must be directly proportional, that is,

$$\Omega_\mathrm{m} = \Omega_{\mathrm{m},0}(z+1)^3$$

Similarly, accounting for the change in volume *and* wavelength, the radiation density parameter scales as

$$\Omega_\mathrm{rad} = \Omega_{\mathrm{rad},0}(z+1)^4$$

Example: Radiation to Mass Dominance When (at what redshift) did mass overtake radiation as the dominant source of energy in the universe?

The moment when mass overtook radiation was when the two density parameters were equal. Thus, we equate the previous two expressions:

$$\Omega_{\mathrm{m},0}(z+1)^3 = \Omega_{\mathrm{rad},0}(z+1)^4$$

which gives $z = \Omega_{\mathrm{m},0}/\Omega_{\mathrm{rad},0} - 1$.

Friedmann's Equation Revisited

Recall the cryptic Friedmann Equation from class:

$$H^2 = H_0^2\left[\Omega_{\mathrm{rad},0}(1+z)^4 + \Omega_{\mathrm{m},0}(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda\right]$$

Here the Ω's are density parameters (as measured now) and Ω_k is related to the overall geometry of the universe, being positive for a closed universe, zero for a flat universe, and negative for an open universe. This equation effectively governs the evolution of the universe, provided we can get good measurements on the current parameters. Note also how this equation reflects the time dependence on the different factors. Though now the universe is driven by dark energy, at sufficiently high z, mass catches up, and then later radiation also catches up.

Midterm Returns and Grade Discussion