Week 7 Notes
Astro 1 (Discussion Sections 101 & 102)

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THIS SHEET MAY NOT BE USED DURING EXAM

Administrative Tasks

Attendance Pass around attendance sheet.

Homework Homework 4 has been graded and returned. I hope to have Homework 5 back by Wednesday or Thursday at the latest.

Questions on Current Week's Homework

Midterm Review

Concepts you MUST master

Next week's midterm will not be cumulative in the strict sense, but it will *assume* you know things from the last midterm. As has been evident in the homework, most of the mathematical tools for the class have been developed and continue to come up again and again. Here I present the ideas you should be very familiar with and when they come in handy.

Small Angle Formula

$$D = \frac{d\alpha}{206,265}$$

Remember, α is the number of arcseconds an object a distance d away subtends it its linear width is D. D and d can be measured in whatever units you like, *so long as they match.*

Use this when comparing sizes of objects as they appear in the sky (or at a large distance) with their actual sizes and/or distances.

Newton's Law of Universal Gravitation

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

As is implicit in the formulation of the equation, the force on each object is *the same*, no matter the mass. However, the objects' *acceleration* due to this force will vary inversely with each objects' mass, according to Newton's second law, $F = ma$. Remember that r is the separation distance between the two centers of the objects, and m_1 and m_2 are the masses of the objects. G is just a constant of proportionality to keep units straight and calibrated.

From this law and Newton's second law we can deduce the acceleration due to gravity of a mass near an object:

$$F = m_1 a = G\left(\frac{m_1 m_2}{r^2}\right) \qquad a_g = G\frac{m_2}{r^2}$$

Use this law when you are trying to find a gravitational force or acceleration between two objects.

Newton's Form of Kepler's Third Law

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right] a^3$$

This law *always* works. The condensed form of the law, $P^2 = a^3$ is only valid for objects orbiting the sun *and* when the period P is measured in years and the semi-major axis a is measured in astronomical units (AU).

Newton's formulation of the law requires you to use SI units for every quantity. Remember that a is the semi-major axis of the orbit connecting the two *centers* of the objects, measured in meters. P is the period it takes for one orbit to be completed, and by default it is measured in seconds. Look for times when one mass is much larger than the other, and then $m_1 + m_2$ can be approximated by simply m_1 or m_2, whichever is taken to be the larger mass. For example, a satellite orbiting Earth: the satellite is MUCH less massive than earth, so its mass is approximately zero. Also, be sure that for close orbits, you keep track of both the distance to the surface of the object *and* the radius of the object. In the satellite example, this means that $a =$ (distance from surface of earth to satellite) + (radius of Earth).

Use this law when you are dealing with orbits of systems. Typically when you are looking for a period, mass, or semi-major axis, this equation is the weapon of choice.

Speed of Light

$$c = \lambda\nu$$

Here c is the speed of light, typically given as $c = 3.0 \times 10^8$ m/s. Then λ is the wavelength of the light, measured in meters and ν is the frequency of the light, measured in Hz (Hertz), which is the same as 1/s.

Use this law when converting between wavelength and frequency of light.

Energy of a photon

$$E = h\nu = \frac{h\lambda}{c}$$

Here h is Planck's constant and E is the energy of the photon, measured in Joules (J=N·m=kg m^2/s^2). Note that we can convert from one form of the energy to the other by making use of the speed of light equation above.

Use this law when finding the energies of photons or discerning the wavelength and/or frequencies from the known energy of a photon.

Blackbody Laws First we state Wien's Law:

$$\lambda_{\text{max}} = \frac{0.0029\,\text{K m}}{T}$$

Here λ_{max} is the peak wavelength of a blackbody spectrum, measured in meters. That is, if we measured the intensities of light at different wavelengths, λ_{max} would have the strongest intensity. Here, K means Kelvins (not a variable) and m means meters (not a variable), and T is the surface temperature of the object, measured in Kelvin.

Use this law to find the surface temperature of an object given its peak wavelength from blackbody radiation (or vice versa).

Now we state the Stefan-Boltzmann Law for Blackbodies

$$F = \sigma T^4$$

Here F is the Energy Flux, typically measured in $\mathrm{W/m^2}$ ("Watts per square meter"). This is the amount of electromagnetic energy passing through a square meter in one second *on the surface of the object*, and can usually be expressed as $F = P/A$ where P is the power generated by the object (Energy emitted per second) and A is the total area through which the power is passing. σ is a constant of proportionality, and T is again the surface temperature of the object.

Use this law to find the surface temperature given *surface* flux (you may need to convert flux from one location to the surface by first finding the power, which is constant in distance from the source). You can also use this to find surface flux given surface temperature. Then the flux at any distance can be determined by first going through the power.

Doppler Effect

$$\frac{\Delta\lambda}{\lambda_0} = \frac{\nu}{c}$$

Here $\Delta\lambda = \lambda - \lambda_0$ is the apparent shift in wavelength from the the "true" wavelength of light, measured in whatever units you like, so long as they are the same as those of λ_0, which is the "true" wavelength of the light. v is the relative speed of the source (with respect to the viewer), measured in m/s, and c is the speed of light. v is *positive* when an object is moving *away* and *negative* when an object is moving *closer*.

The applications of this law are usually pretty obvious. It is used typically to find the speed of an object given a redshift or blueshift condition.

Diffraction-limited Angular Resolution

$$\theta = 2.5 \times 10^5 \frac{\lambda}{D}$$

Here, θ is the smallest possible angle a telescope can resolve (see clearly) measured in arcseconds, given the wavelength of the light it is receiving λ (in meters) and the diameter of the objective lens of the telescope, D, which is measured in meters.

Again, the context of a problem usually makes the use of this equation obvious. It only deals with the resolving power of telescopes.

Escape Velocity

$$v_{\mathrm{esc}} = \sqrt{\frac{2GM}{R}}$$

v_{esc} is the speed an object must attain to fully escape the gravitational pull of an object of mass M (measured in kg) and radius R, measured in meters. Again, G is the gravitational constant.

Average speed of a gas particle

$$v_{\mathrm{avg}} = \sqrt{\frac{3kT}{m}}$$

v_{avg} is the average speed of a gas particle of mass m (in kg) in an overall gas of temperature T (in Kelvins). Here k is Boltzmann's constant.

Requested Examples/Explanations

Test Tips

"Problems" To solve problems requiring different laws, try to identify what the relevant quantities are to pick what subject you're dealing with. If you notice that you have the angular size of an object and its distance from us, perhaps start thinking about using the small angle formula to see if it gives you any useful information. To prepare for the test, I have two suggestions:

1. Look at the "boxes" in the textbook... all of them. They typically do a good job of explaining the equations at hand and they work good examples. If the examples don't make sense, try copying each step down and convincing yourself that it makes sense. Do this until you can do it without the book and have a good sense of what's going on.

2. Look at the homework solutions (or your homework, if it's done well). These are good examples of the kinds of questions that may be asked on an exam.

Conceptual Questions This review has focused more on problem-solving, but as you know, that is only part of what you will be tested on. Look over relevant concepts in your notes and textbook. Ask yourself, "What causes global warming?" or "Why does Mars have such a thin atmosphere?" or perhaps, "What does an absorption spectrum tell astronomers?" The review questions at the end of the chapter are very helpful in posing relevant questions that may translate well into concepts you will be tested on.

Still Confused? If you are trying to solve a problem and are just stuck, or can't understand how the book (or I) did a step of a problem, or perhaps the phrase "plate tectonics" sends a shiver down your spine, come see me in office hours and I'd love to try and help you. Is it Thursday night and you're regretting not having prepared earlier when a difficulty strikes you? Send me an e-mail with "`[ASTRO I]`" in the subject and I'll do my best to get back to you (I'm usually up all night any way doing homework).

Other Questions

Good luck on your exam!