Week 5 Notes Astro 1 (Discussion Sections 101 & 102)

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Administrative Tasks

Stargazing Sign-In Last week a sign-up sheet was circulated regarding the stargazing night. If you missed out and are interested in going, please e-mail me.

Website Reannounce website, which is www.physics.ucsb.edu/~wmwolf. Relevant files are under the "teaching" tab.

Test Results Test results were handed out last week. If you were not here and signed permission to have tests returned in public, your results are where your homework is normally returned. Otherwise I still have it. Curve was RawScore/28 * $1.1 \times 100\%$. The average test score was around 17.5/30.

Homework Many mistakes are being made due to neglecting the use of units. While I will only take points off for not having units on your final answer, you get a good check on your answer by evaluating your units as you go. This is called **dimensional analysis**.

Also, the answers in the back of the book are strictly a *reference*, not a goal to work towards. Many (and I mean *many*) students had completely incoherent work that led to bad answers. A result given in meters would have an arrow that pointed to the correct answer expressed in minutes. This is totally unacceptable and will gain you no points. It insults your graders' intelligence and will only hurt your grade. Work towards your own answer and check its validity with the provided one. You are better off doing honest work and evaluating your answer to see if it even makes physical sense rather than doing nonsense work and vomiting out the right answer.

Questions

Concept Review

Newton's Form of Kepler's Third Law While we have already used this law to solve problems, it's worth revisiting. Originally, Kepler's Third Law essentially stated

$$P^2 = ka^3$$

with P the orbital period of two bodies, a the semimajor axis of the orbit, and k some proportionality constant. With his laws of motion in place, Newton modified Kepler's third law to include more valid information. It's restatement is given as

$$P^2 = \left[\frac{4\pi^2}{G(m_1 + m_2)}\right]a^3$$

where P is the orbital period, m_1 and m_2 are the masses of the two objects, G is the gravitational constant, and a is the semi-major axis connecting the two *centers of mass*. Essentially Newton found the k that was somewhat mysterious in Kepler's original formulation. With this in place, we can now find out information about objects orbiting things other than the sun. **Average Density** The density of an object measures how closely packed the matter is in a given object. For instance, the matter of a bag of popcorn is packed in less densely than a bag of coins. Thus, the bag of coins is heavier, even if it has the same volume. The density of an object is typically represented by the Greek letter, ρ ("rho"), and the **average density** is calculated thusly:

$$\rho_{\rm avg} = \frac{M}{V}$$

where M is the mass of the object and V is the volume. For a sphere then,

$$V = \frac{4}{3}\pi R^3$$

Given the mass of the sphere, we can calculate its average density pretty easily.

Escape Velocity The escape velocity of an object is the velocity required to completely break free of the object's gravitational pull. This can be derived fairly easily by noting that the potential and kinetic energies are defined to be

$$E_p = -G\frac{Mm}{r}$$
$$E_k = \frac{1}{2}mv^2$$

Total energy is conserved, so $E_k + E_p = E_{tot} = a$ constant. Escaping gravitational pull completely essentially means that you are at an infinite distance from the object. We see that as our distance goes to infinity, potential energy goes to zero. The minimum amount of energy then (after escaping) is zero. So, let us set E_{tot} to zero and find what velocity is required when we start at the surface of the object r = R.

$$0 = E_{\text{tot}}$$
$$= E_k + E_p$$
$$= \frac{1}{2}mv_{\text{esc}}^2 - G\frac{Mm}{R}$$
$$G\frac{Mm}{R} = \frac{1}{2}mv_{\text{esc}}^2$$
$$\frac{2GM}{R} = v_{\text{esc}}^2$$
$$\sqrt{\frac{2GM}{R}} = v_{\text{esc}}$$

So as it turns out, the minimum velocity needed to escape gravitational attraction is independent of mass, which is pretty cool! That being said, the acceleration required to get to that mass is directly proportional to mass, as shown by Newton's second law.

Special Case: What happens when $v_{\text{exc}} > c$ (the speed of light)? This is essentially what a black hole is. Since nothing is physically capable of going faster than light, once you are under the influence of a black hole, you cannot escape. This is why black holes are "black," as even light cannot escape the gravitational pull past a specific radius known as the "event horizon."

Example: Escape velocity of Earth What is the escape velocity of earth? Earth's radius is 6,378 km and its mass is 5.74×10^{24} kg.

We simply apply the formula we derived for escape velocity (note how the units work out!):

$$v_{\rm esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67 \times 10^{-11} \,\mathrm{Nm}^2/\mathrm{kg}^2)(5.74 \times 10^{24} \,\mathrm{kg})}{6378 \,\mathrm{km}}} = 11,000 \,\mathrm{m/s} = 11 \,\mathrm{km/s}$$

Thermal Energy of Particles Temperature is an indicator of random motions of particles. This energy is completely random, and is due only to the kinetic energy of the particles. In equation form, it is given by

$$E_k = \frac{3}{2}kT$$

where k is Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K and T is the temperature of the system in Kelvins.

Example: Speed of oxygen molecules at room temperature given that the average oxygen molecule has a mass of 5.32×10^{-26} kg.

We simply apply our two equation for kinetic energy of such a system:

$$E_k = \frac{3}{2}kT = \frac{1}{2}mv^2$$

Rearranging, we have then

$$v^2 = \frac{3kT}{m}$$

and then

$$v=\sqrt{\frac{3kT}{m}}$$

Suppose room temperature is around $20^{\circ} \text{C} = 293 \text{ K}$, then from plugging everything in, we get v = 478 m/s. Note again that the units work out nicely, since $1 \text{ J}=1 \text{ N}\cdot\text{m}=1 \text{ kg} \text{ m}^2/\text{s}^2$.

Atmospheric Escape: In the previous example, we found the *average* speed of O_2 molecules in the room. Actual velocities can vary greatly, and in fact, at every moment, particles that have attained velocities greater than the escape velocity of Earth go careening out of our atmosphere. This process never stops, as these molecules continually get "thermal kicks" from collisions with each other, sunlight, and other energy sources, so why do we still have an atmosphere after five billion years?

This is actually a fairly troubling question, because to sustain life, a planet must obviously have an atmosphere. There must be a source, then, to keep gas particles around the planet. In most cases, this source is geologic. Volcanos and other features spew all sorts of gases into the air which are then broken down and reform into the atmosphere we have today. Thus, when Earth ceases to be geologically active, our atmosphere will begin a slow decline.

Perhaps I'm just making this up, you say, but we already have evidence of this happening: Mars. Mars has known extinct volcanoes and a lot of indicators that it once had a decent atmosphere, and even large water sources (you'll study these soon enough). However, since Mars is significantly smaller than earth, its geologic activity halted a long time ago. Due to its smaller mass, its escape velocity is much lower than that of Earth, so gas particles readily escaped with none to replace them. Now Mars is a desolate place with a very thin atmosphere. What's left is a shadow of its former self. The only water and other atmospheric gases are locked up underground, frozen, or on the polar caps.